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# Poverty Measurement and Policy: From FGT to the MPI

James E. Foster

George Washington University and OPHI, Oxford

G20 T20 TF5 Side Event

“Multidimensional Poverty in the Midst of the COVID-19 Pandemic: A  
Commitment to Reducing Poverty in All Its Forms”

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# Introduction

Universally agreed

Indonesia has high **statistical capacity**

See *Measuring the Statistical Capacity of Nations*, Oxford Bulletin of Economics and Statistics, 2021

Reflected in national poverty statistics

Quality due to the good work of many at this event

Adheres to World Bank standards for consumption poverty

Rich, frequently gathered data sources; unified data

Frequent reporting of FGT measures to understand incidence, depth, and severity of poverty

Note Uses full spectrum of FGT's powers and properties

Decomposability – Connect up local with national poverty

Simplicity – move beyond headcounts intuitively

Other helpful axioms and characteristics

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# Introduction

Recently Alkire-Foster (2011) extended FGT to multidimensional poverty

This new technology was soon adopted by

UN in its Global MPI; a bit later by WB in MPM

Many nations as they grapple with SDG target 2.1

Today's focus

What is the AF approach to poverty?

How does it relate to the FGT measures?

How might it augment the view offered by consumption poverty?

How might it help direct policy?

Bottom line: MPI is as simple as the poverty gap, but is a far more powerful tool for directing policies

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# Introduction

Outline (remembering time limit!)

Quick review of FGT

Motivation, definitions, axioms, applications

Brief presentation of AF

Motivation, definitions, axioms, applications

Discussion of possibilities

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# General Poverty Measurement

Traditional framework of Sen (1976)

Two steps

Identification: “Who is poor?”

Targeting

Aggregation: “How much poverty?”

Evaluation and monitoring

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# Consumption Poverty Measurement

Uses **poverty line** for identification

Poor if consumption below the cutoff

Example: distribution  $x = (7,3,4,8)$  poverty line  $\pi = 5$

Who is poor?

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# Consumption Poverty Measurement

Uses **poverty measure** for aggregation

The FGT family is defined as

$$P_a(x; \rho) = m(g_1^a, \dots, g_n^a) = m(g^a)$$

where  $x = (x_1, \dots, x_n)$  is the distribution of consumption.

$\pi$  is the poverty line, and

$g_i^a$  is  $[(\pi - x_i)/\pi]^a$  if  $i$  is poor and 0 if  $i$  is not poor

Key measures

$P_0$  is headcount ratio

$P_1$  is per capita poverty gap

$P_2$  is squared gap

# Consumption Poverty Measurement

Example

**Distribution**  $x = (7, \underline{1}, \underline{4}, 8)$

**Poverty line**  $\pi = 5$

**Deprivation vector**  $g^0 = (0, 1, 1, 0)$

**Headcount ratio**  $P_0(x; \pi) = \mu(g^0) = 2/4$

**Normalized gap vector**  $g^1 = (0, 4/5, 1/5, 0)$

**Poverty gap = HI =**  $P_1(x; \pi) = \mu(g^1) = 5/20$

**Squared gap vector**  $g^2 = (0, 16/25, 1/25, 0)$

**FGT Measure =**  $P_2(x; \pi) = \mu(g^2) = 17/100$

Note that **FGT** or  $P_\alpha$  is the mean of a vector



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# Consumption Poverty Measurement

What properties are satisfied by FGT?

Digression: rethinking the role of axioms

Old view: technical requirements

New view: nuggets of policy

Categories

Invariance axioms

Concern changes in the distribution should be ignored by the measure

Dominance axioms

Concern changes in the distribution that decrease poverty

Composition axioms

Specifies the impact of combining populations

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# Consumption Poverty Measurement

## List of axioms for FGT

For  $\alpha = 0$  (headcount ratio)

Invariance Axioms: Symmetry, Replication Invariance, Focus

Composition Axioms: Subgroup Consistency, **Decomposability**

For  $\alpha = 1$  (poverty gap)

+Dominance Axiom: **Monotonicity**

For  $\alpha = 2$  (FGT)

+Dominance Axioms: Transfer

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# Multidimensional Poverty Measurement

How to measure poverty when there are many dimensions?

Alkire and Foster (2011):

Based the **aggregation** step on FGT generalized to many dimensions

Proposed a new dual cutoff approach to **identification**

**Deprivation cutoffs**  $z_1 \dots z_j$  one per each of  $j$  deprivations

**Poverty cutoff**  $k$  across aggregate weighted deprivations

Idea

A person is poor if multiply deprived enough

Example

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# AF Methodology

## Achievement Matrix

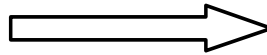
$$Y = \begin{matrix} & \mathbf{Dimensions} & & & & \\ & & & & & \mathbf{Persons} \\ \begin{matrix} 13.1 & 14 & 4 & 1 \\ 15.2 & \underline{7} & 5 & \underline{0} \\ \underline{12.5} & \underline{10} & \underline{1} & \underline{0} \\ 20 & \underline{11} & 3 & 1 \end{matrix} & & & & & \\ \mathbf{z} = (13 & 12 & 3 & 1) & & \mathbf{Cutoffs} \end{matrix}$$

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# AF Methodology

Deprivation Matrix

$$g^0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 2 \\ 4 \\ 1 \end{bmatrix}$$



Censored Deprivation Matrix,  $k=2$

$$g^0(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 2 \\ 4 \\ 0 \end{bmatrix}$$

**Identification** Who is poor?

If poverty cutoff is  $k = 2$

Then the two middle persons are poor

Now censor the deprivation matrix

Ignore deprivations of nonpoor

# AF Methodology

If data cardinal, construct two additional censored matrices

Censored Gap Matrix

$$g^1(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42 & 0 & 1 \\ 0.04 & 0.17 & 0.67 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Censored Squared Gap Matrix

$$g^2(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.42^2 & 0 & 1^2 \\ 0.04^2 & 0.17^2 & 0.67^2 & 1^2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

**Aggregation**

$$M_{\alpha} = \mu(g^{\alpha}(k)) \text{ for } \alpha \geq 0$$

**Adjusted FGT**  $M_{\alpha}$  is the mean of the respective censored matrix

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# AF Methodology

## However

The poverty measures with  $\alpha > 0$  use gaps, hence require **cardinal** data

Impractical given data quality

Focus has been on  $\alpha = 0$  that handles **ordinal** data

## **Adjusted Headcount Ratio $M_0$ or MPI**

Practical and applicable

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# Adjusted Headcount Ratio

$$\text{Adjusted Headcount Ratio} = M_0 = HA = \mu(g^0(k))$$

	Domains	c(k)	c(k)/d	
$g^0(k) =$	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	$\mathbf{0}$		
		$\underline{\mathbf{2}}$	$\mathbf{2 / 4}$	Persons
		$\underline{\mathbf{4}}$	$\mathbf{4 / 4}$	
		$\mathbf{0}$		

H = multidimensional headcount ratio = 1/2

A = average deprivation share among poor = 3/4



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# Adjusted Headcount Ratio

## Axioms

Invariance Axioms: Symmetry, Replication Invariance,  
Deprivation Focus, Poverty Focus

Dominance Axioms: Weak Monotonicity, **Dimensional Monotonicity**, Weak Rearrangement, a form of Weak Transfer

Composition Axioms: Subgroup Consistency, **Decomposability**, Dimensional Breakdown

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# THE MPI AND THE POVERTY GAP

$$\text{MPI} = \text{M}_0 = \text{H} \times \text{A}$$

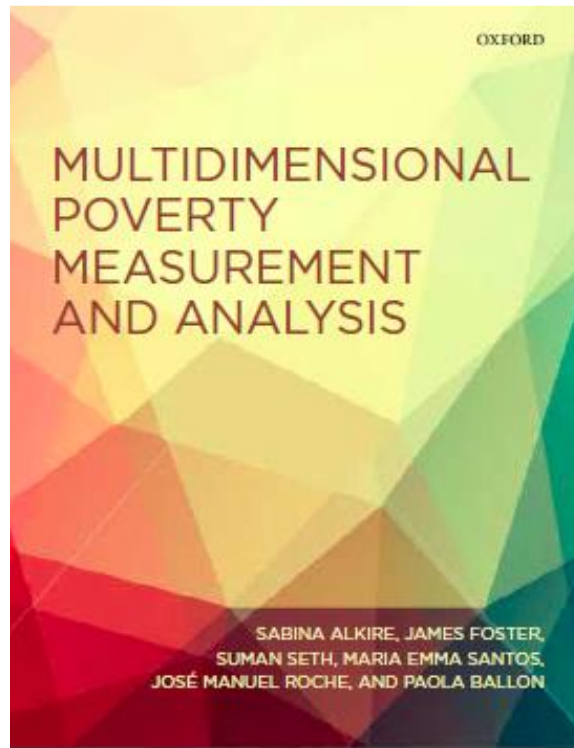
H = Multidimensional poverty rate

A = Average intensity (breadth) of poverty

$$\text{Poverty Gap} = \text{P}_1 = \text{H} \times \text{I}$$

H = Monetary poverty rate

I = Average intensity (depth) of poverty



*Econometrica*, Vol. 52, No. 3 (May, 1984)

## A CLASS OF DECOMPOSABLE POVERTY MEASURES

BY JAMES FOSTER, JOEL GREER, AND ERIK THORBECKE<sup>1</sup>

SEVERAL RECENT STUDIES OF POVERTY have demonstrated the usefulness of breaking down a population into subgroups defined along ethnic, geographical, or other lines [e.g. 1, 20]. Such an approach to poverty analysis places requirements on the poverty measure in addition to those proposed by Sen [15, 16]. In particular, the question of how the measure relates subgroup poverty to total poverty is crucial to its applicability in this form of analysis. At the very least, one would expect that a decrease in the poverty level of one subgroup *ceteris paribus* should lead to less poverty for the population as a whole. At best, one might hope to obtain a quantitative estimate of the effect of a change in subgroup poverty on total poverty, or to give a subgroup's contribution to total poverty.

One way to satisfy the above criteria is to use a poverty measure that is additively decomposable in the sense that total poverty is a weighted average of the subgroup poverty levels.<sup>2</sup> However, the existing decomposable poverty measures are inadequate in that they violate one or more of the basic properties proposed by Sen.<sup>3</sup> Stated another way, of all the measures [1, 3, 10, 19] that are acceptable by the Sen criteria, none is decomposable. In fact, the Sen measure and its variants that rely on rank-order weighting fail to satisfy the basic condition that an increase in subgroup poverty must increase total poverty (see footnote 6). This note is a first step towards resolving these inadequacies.

In what follows we present a simple, new poverty measure<sup>4</sup> that (i) is additively decomposable with population-share weights, (ii) satisfies the basic properties proposed by Sen, and (iii) is justified by a relative deprivation concept of poverty. The inequality measure associated with our poverty measure is shown to be the squared coefficient of variation and indeed the poverty measure may be expressed as a combination of this inequality measure, the headcount ratio, and the income-gap ratio in a fashion similar to Sen [15]. We generalize the new poverty measure to a parametric family of measures where the parameter can be interpreted as an indicator of "aversion to poverty." A brief empirical application demonstrates the usefulness of the decomposability property.

### 1. A DECOMPOSABLE POVERTY MEASURE

Let  $y = (y_1, y_2, \dots, y_n)$  be a vector of household incomes in increasing order, and suppose that  $z > 0$  is the predetermined poverty line. Where  $g_r = z - y_r$  is the income shortfall of the  $r$ th household,  $q = q(y; z)$  is the number of poor households (having income no greater than  $z$ ), and  $n = n(y)$  is the total number of households, consider the

<sup>1</sup>We would like to thank the participants of the Cornell Development Seminar, Gary Fields, and the anonymous referees for helpful comments. In addition, we owe a special debt of gratitude to Amartya Sen for his thoughtful remarks and encouragement. This note is based on a longer working paper [8] and on dissertation research by J. Greer.

<sup>2</sup>See [1, 20]. In contrast, decomposability as applied to inequality measures involves a "between-group" term to account for differences among subgroup mean incomes [4, 17]. Here one poverty level is postulated to apply to all subgroups; hence a "between-group" poverty term would appear to be unnecessary.

<sup>3</sup>In their empirical work, Anand [1], Kakwani [9], and Van Ginneken [20] use decomposable measures that violate the transfer axioms.

<sup>4</sup>While revising the initial submitted version, we became aware of independent work by Kundu [12] which also gives  $P_2$  and indicates some of its properties. However, Kundu's paper addresses quite different issues and, in particular, decomposability is not mentioned.

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# Who Is Using MPIs?

## Governments

Many national measures, including India; MPPN 60 countries

## United Nations

2010 Global MPI

2015 Sustainable Development Goals (**SDGs**)

Target 1.2: by 2030, reduce at least by half the proportion of men, women and children of all ages living in **poverty in all its dimensions** according to national definitions

## World Bank

2016 Poverty Commission (**Atkinson Report**)

Recommendation 19: The Complementary Indicators should include a multidimensioned poverty indicator based on **the counting approach**.

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# Rationale for Using MPIs

Transparent **headline** metric for communication/monitoring

**Simple** yet sophisticated

Linked to well-embedded **FGT**

Coordinated **dashboard** for policy analysis

**Decomposability** axiom links overall to local

**Dimensional Breakdown** axiom reveals structure of poverty

Ex: Global MPI across Africa

Supports good **governance**

Consistent with **political cycle** – can observe results

Reduces **political risk** from income shocks (eg, Mexico)

**Aligns** government and citizen

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# Rationale for Using MPIs

Can help **coordinate** national development policy

**De-siloing** government ministries (Ex: Colombia)

“Measurement as leadership”

Guiding development **budgets** (Ex: Costa Rica)

**Upgradeable** technology to account for inequality

If data are cardinal – **M1** or **M2** (captures inequality)

If ordinal – **M-gamma** measures

Fully **localizable**

Global MPI – regional MPIs – National MPIs – Local MPIs

What do you think? Could these rationales apply here?

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Thank you!

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